

# Understanding PMU data

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## Types of grid measurement data

Advanced Metering Infrastructure (AMI)

*kW and kWh consumption at customer meters, typically reported at 15-min resolution*

Supervisory Control and Data Acquisition (SCADA)

*Voltage or current magnitudes, reported at resolution on the order of several seconds*

Phasor Measurement Units (PMUs)

*Voltage or current magnitudes and phase angles, frequency and derivative quantities, reported roughly each cycle (25-120 Hz)*

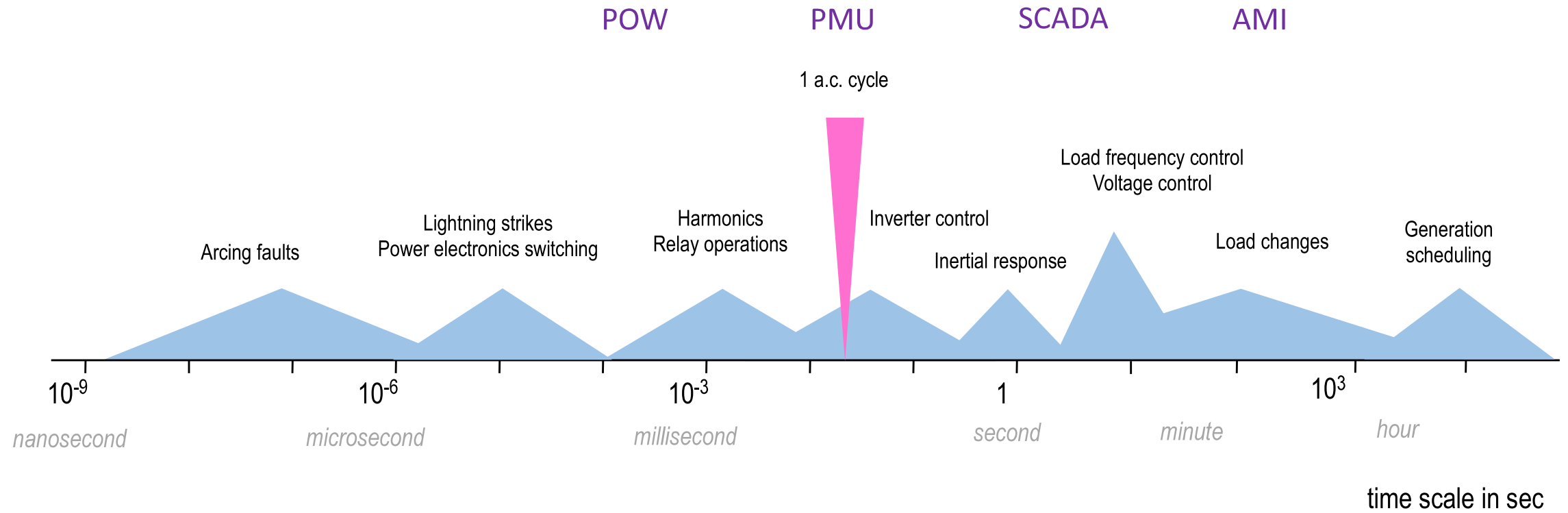
Event-triggered Point-on-Wave (POW)

*256 to 1 million samples/sec of voltage or current waveform, reported for a short duration or on a continuous monitoring basis*

Continuous Point-on-Wave (CPOW)

For more, see: A. Silverstein and J. Follum: High-Resolution, Time-Synchronized Grid Monitoring Devices  
[https://naspi.org/sites/default/files/reference\\_documents/pnnl\\_29770\\_naspi\\_hires\\_synch\\_grid\\_devices\\_20200320.pdf](https://naspi.org/sites/default/files/reference_documents/pnnl_29770_naspi_hires_synch_grid_devices_20200320.pdf)

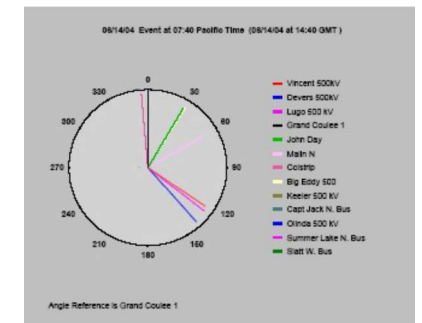
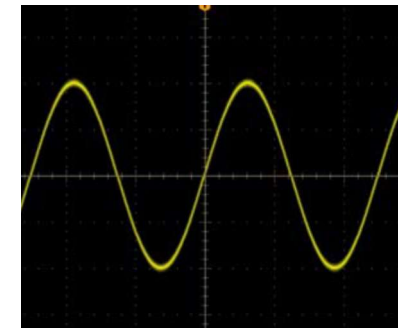
# Time scales for electric grid events and control

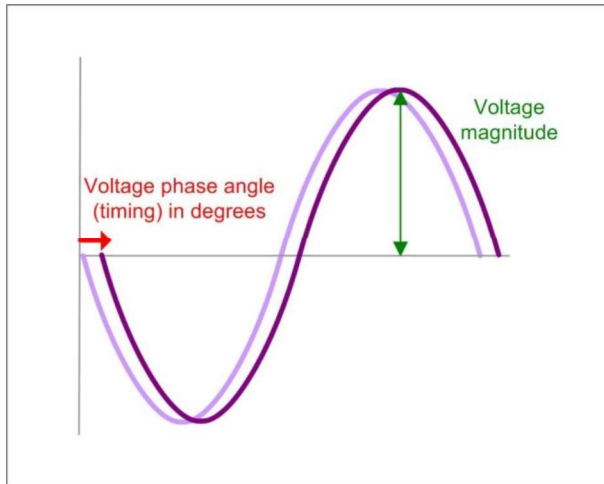


# Phasor Measurement Data a.k.a. Synchrophasors

*Phasor measurements of alternating current or voltage quantities give an abstract image of what is happening physically, based on an implicit model.*

*The phasor representation is synthesized from (many) raw analog measurements in a lossy compression algorithm.*





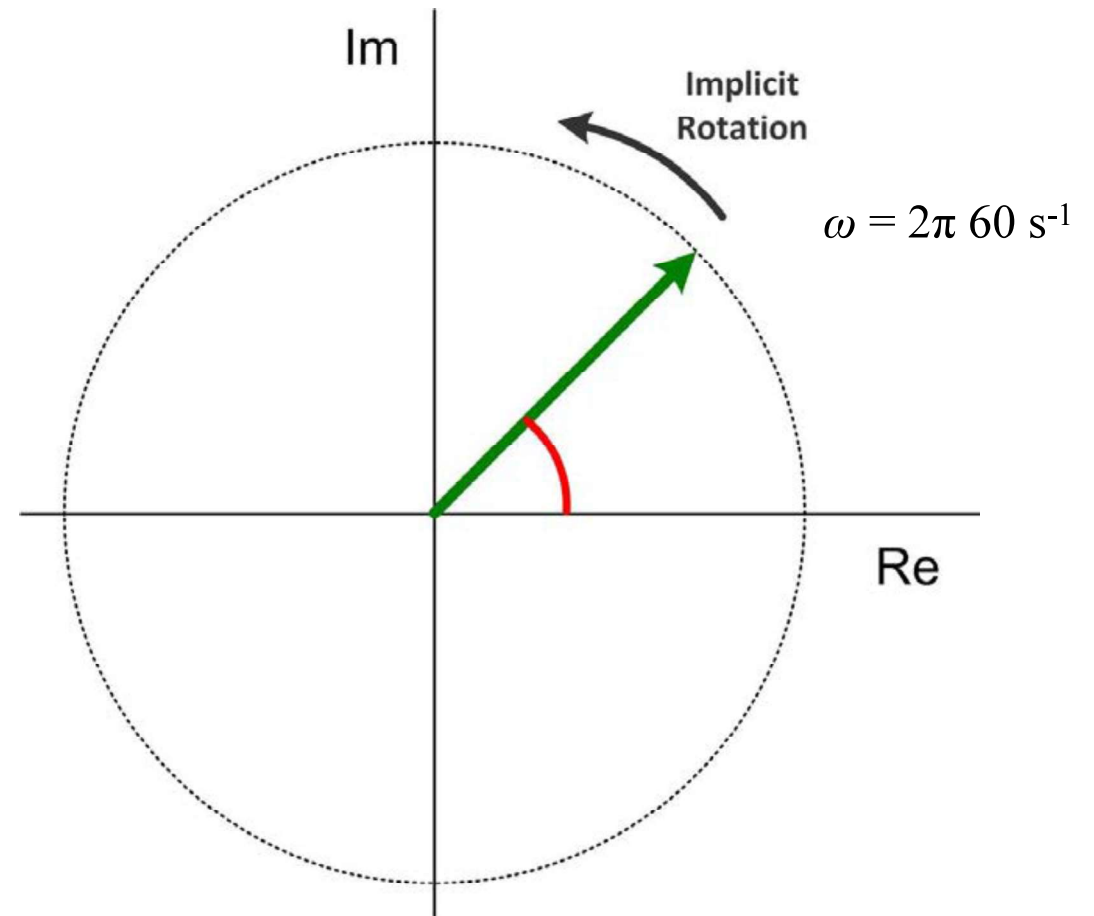
## Time Domain vs. Phasor Domain

*The sinusoidal function in the time domain*

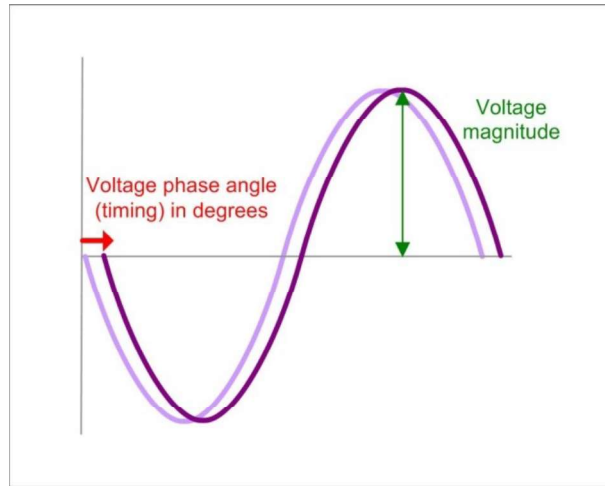
$$v(t) = V_{max} \cos(\omega t + \phi)$$

*is written as the phasor*

$$\mathbf{V} = |V| e^{j\phi}$$



## Getting to Phasor Notation



We assume a waveform is represented by a pure sinusoid:

$$v(t) = V_{max} \cos(\omega t + \phi)$$

Now we imagine that we are looking at the real part of a complex quantity

$$v(t) = \text{Re}\{\mathbf{v}(t)\}$$

where  $\mathbf{v}(t) = \cos(\omega t + \phi) + j \sin(\omega t + \phi)$

$$e^{jx} = \cos x + j \sin x$$

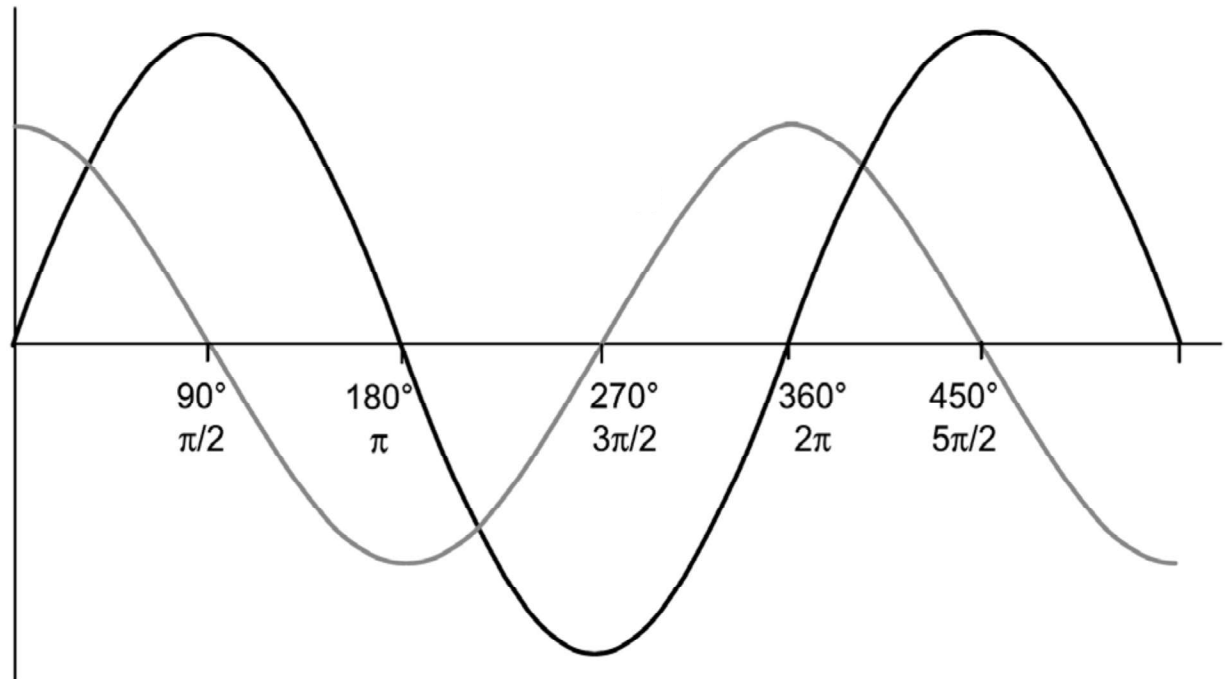
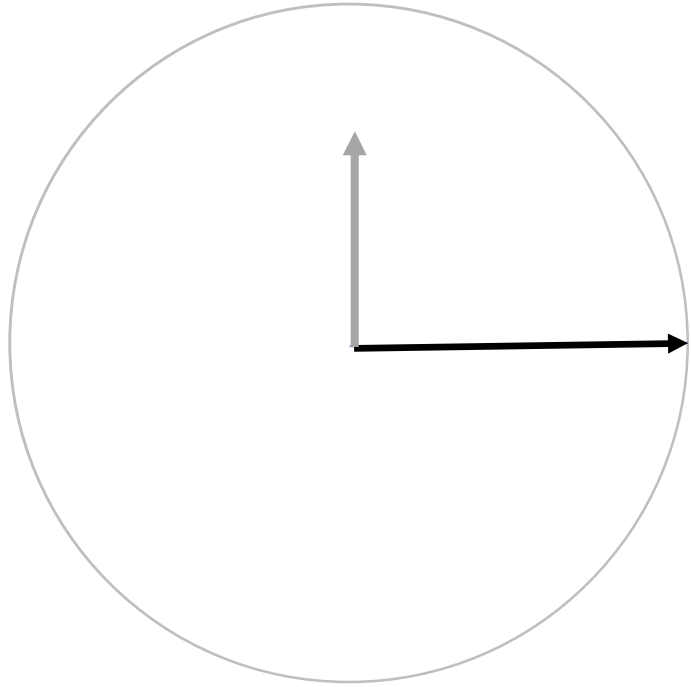
Using Euler's equation, we format this as a complex exponential:

$$v(t) = \text{Re}\{V_{max} e^{j(\omega t + \phi)}\} = \text{Re}\{V_{max} e^{j\omega t} e^{j\phi}\}$$

For steady-state analysis, assuming we already know everything about frequency, we discard the “rotating phasor” and keep only the “stationary” exponential term with the phase angle:

$$v(t) = \cos(\omega t + \phi) \Rightarrow \mathbf{V} = V_{rms} e^{j\phi} = V_{rms} \angle \phi$$

Finally, in power engineering convention, we use the root-mean-square (rms) magnitude instead of the amplitude.



*Displaying waves as a phasor snapshot in time allows easy comparison – assuming everything is at the same frequency!*

$$v_1(t) = V_{1,max} \cos(\omega t + \phi_1)$$

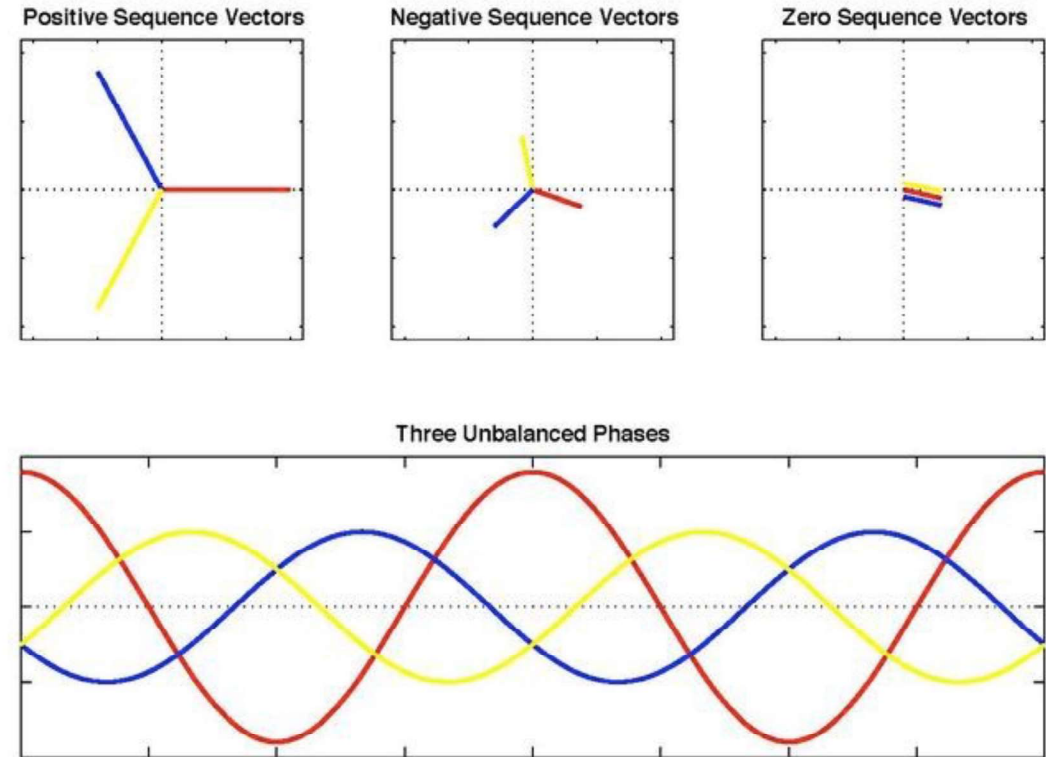
$$v_2(t) = V_{2,max} \cos(\omega t + \phi_2)$$

## Positive-sequence components

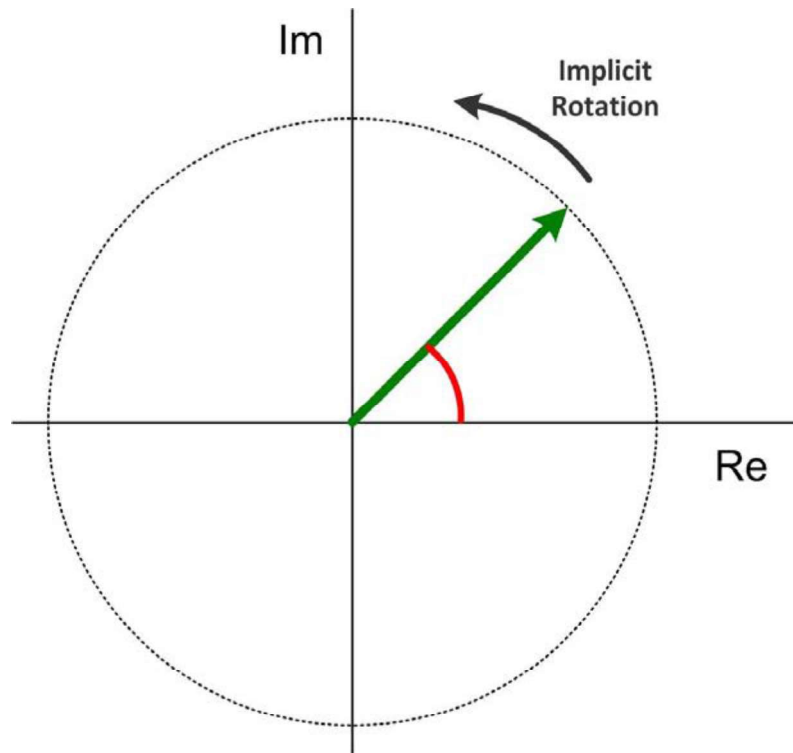
*PMUs may report positive-sequence values rather than all three ABC phases.*

*This contains all the information for the three-phase system iff the phases are balanced.*

Symmetrical components: a way to represent imbalanced three-phase voltages or currents as the linear combination (vector sum) of symmetrical +/-/0 phasor trios



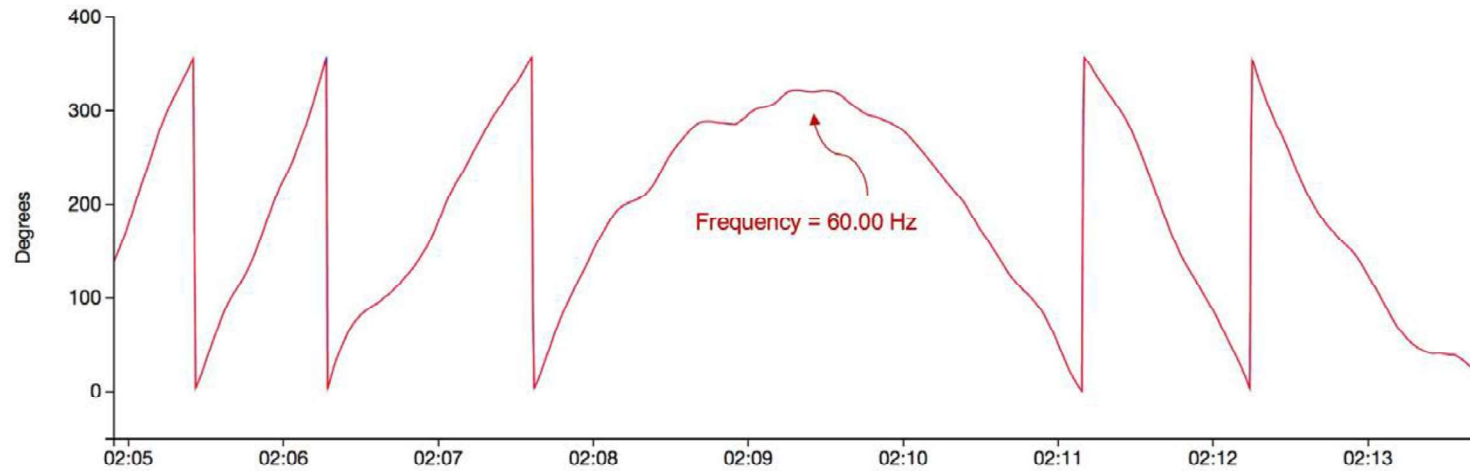




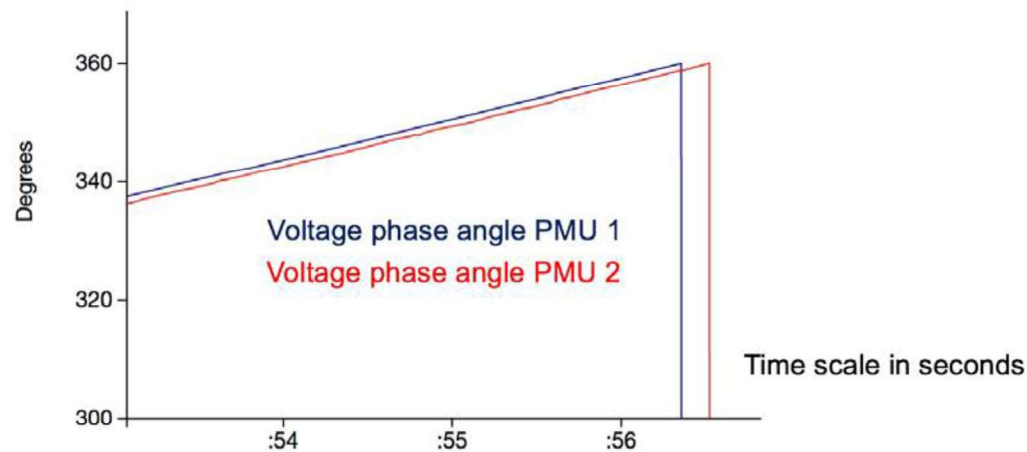
*Measuring relative to a GPS clock, if the a.c. frequency is not exactly constant at 60.000 Hz, we will see the phase angle from a single measurement increasing and decreasing over time, wrapping around from +360 or -360 to 0°*

*In the steady state, phasors have physical meaning only as a difference between two locations.*

## Phasors on time-series display



*Measuring relative to a GPS clock, if the a.c. frequency is not exactly constant at 60.000 Hz, we will see the phase angle from a single measurement increasing and decreasing over time, wrapping around from +360 or -360 to 0°*



*In the steady state, phasors have physical meaning only as a difference between two locations.*

## Building some physical intuition

- AC frequency is (approximately) the same everywhere across a synchronous grid
- Synchronicity comes from rotating machines, electromagnetically coupled
- Imbalances in power generation vs. load make system frequency increase or decrease
- Local power injection or withdrawal can be visualized like a twist on a common rotating shaft
- Torque or twist drives power across the common shaft, analogous to voltage phase angle

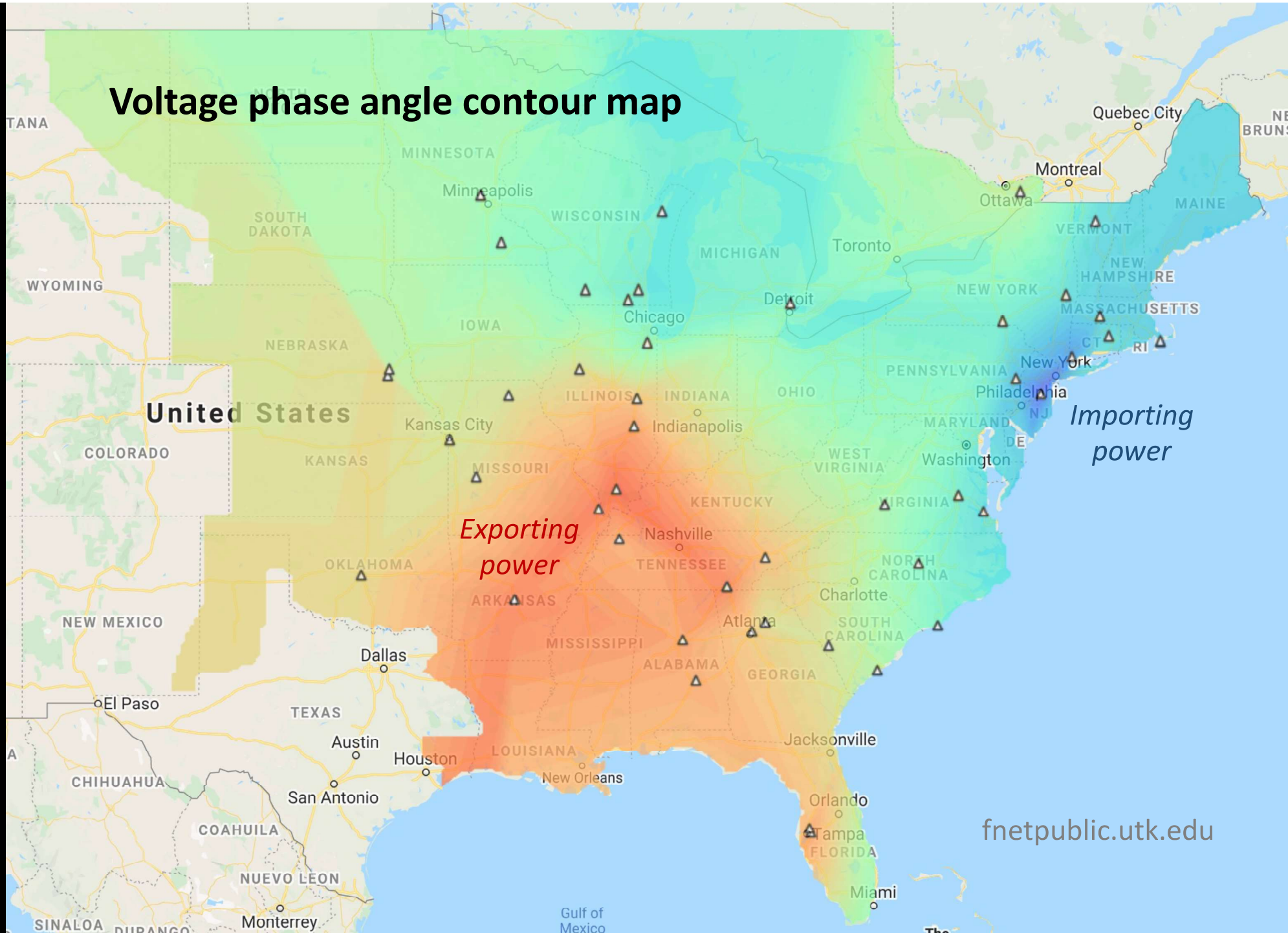




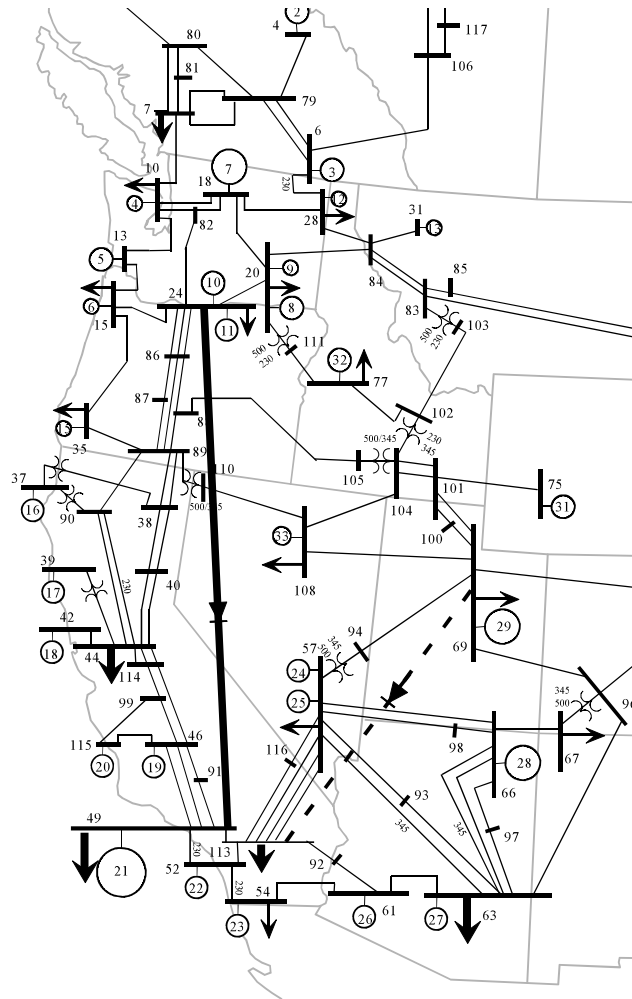


Voltage phasor differences drive power flow across the grid

# Voltage phase angle contour map



# Voltage phasors are the state variables for the power network



*Real and reactive power  $P_i$  and  $Q_i$  at the  $i^{\text{th}}$  bus are determined by all the  $V$ 's and  $\delta$ 's*

$$P_i = \sum_{k=1}^n |V_i||V_k|[g_{ik}\cos(\delta_i - \delta_k) + b_{ik}\sin(\delta_i - \delta_k)]$$

conductance and susceptance of each branch

$$Q_i = \sum_{k=1}^n |V_i||V_k|[g_{ik}\sin(\delta_i - \delta_k) - b_{ik}\cos(\delta_i - \delta_k)]$$

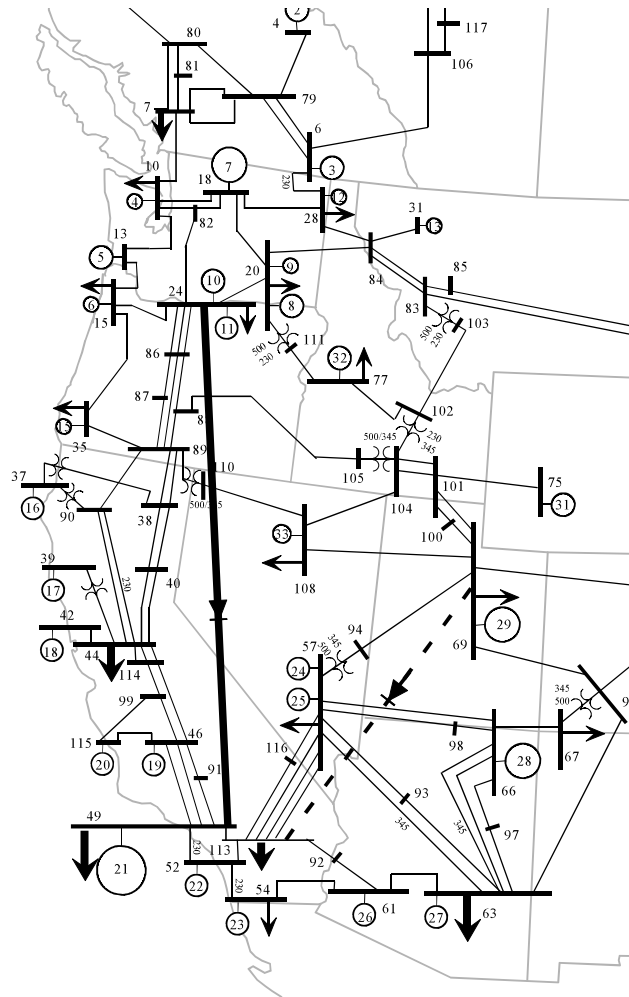
voltage magnitudes

voltage phase angle difference  $\delta_{ik}$

*Power flow across the network is described by a profile of voltage phasors*

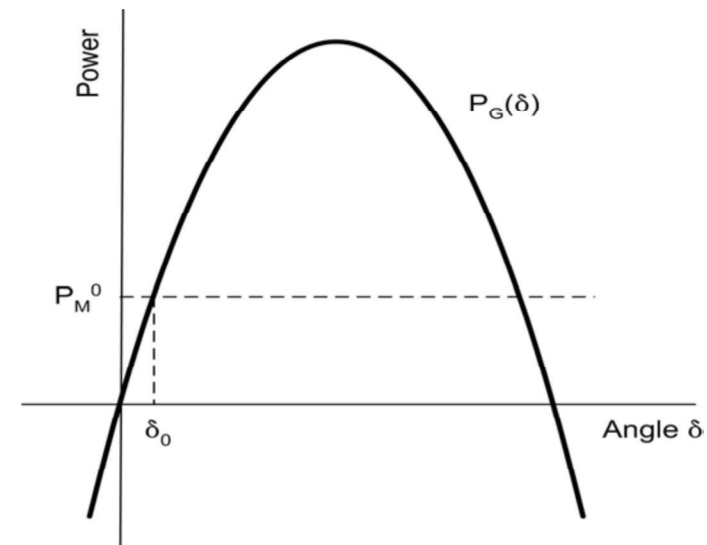
*$P$  depends more on  $\delta$ ,  $Q$  more on  $V$*

# Voltage phasor differences drive power flow across the grid



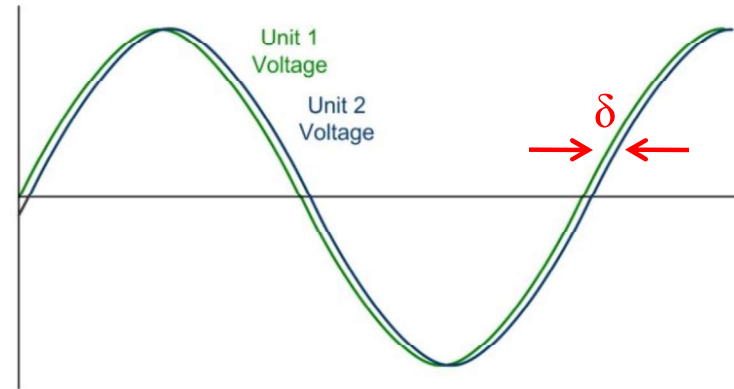
$$P_{12} \approx \frac{V_1 V_2}{X} \sin \delta_{12}$$

*Distance (impedance) matters:  
the farther you transmit a.c. power, the  
larger the voltage phase angle difference,  
and the more wobbly*



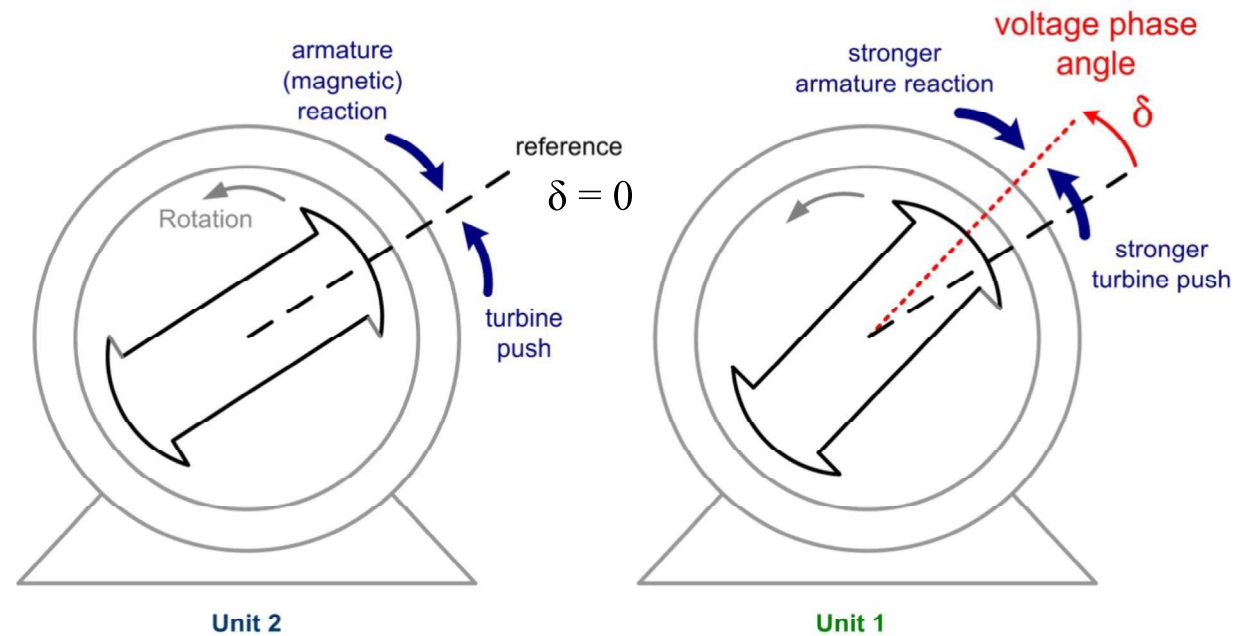


# Voltage phasor differences drive power flow across the grid



*The phase angle difference  $\delta$  between locations drives a.c. power flow*

$$P_{12} \approx \frac{V_1 V_2}{X} \sin \delta_{12}$$

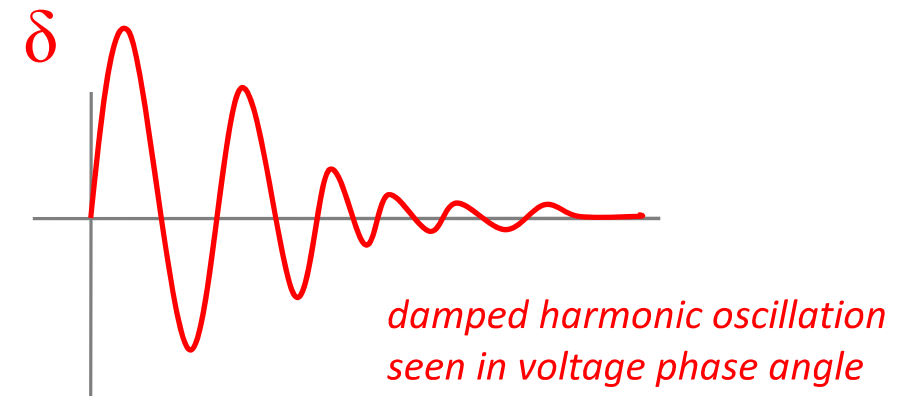
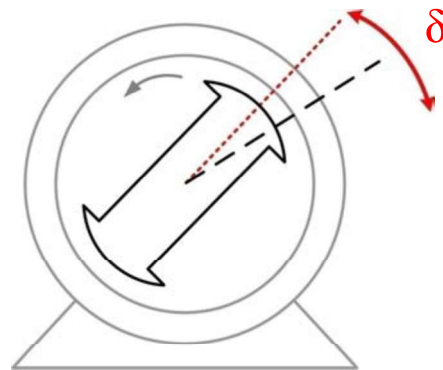


*Power flows from Unit 1 toward Unit 2*



## Thinking about the physics of angle stability

- Power imbalance manifests as a change in angle  $\delta$  and frequency  $\omega$
- Electromagnetic coupling provides negative feedback on rotor position:  
Power injected to the grid by a generator  $P_e$  is a function of  $\delta$
- Rotational inertia tends to hold  $\omega$  steady

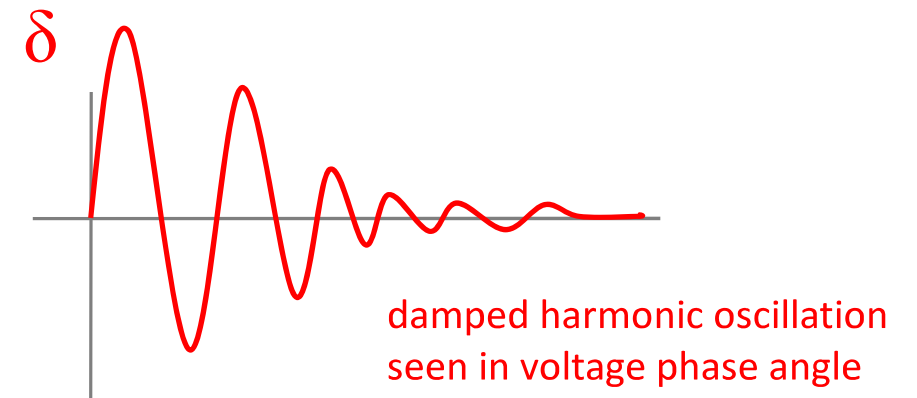
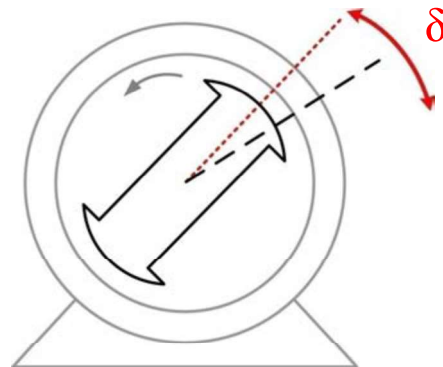


Generator swing equation

$$M \ddot{\delta} + D \dot{\delta} = P_m - P_e(\delta)$$

# Thinking about the physics of angle stability

*Voltage angle is a key variable, but it was not directly observable without PMUs!*

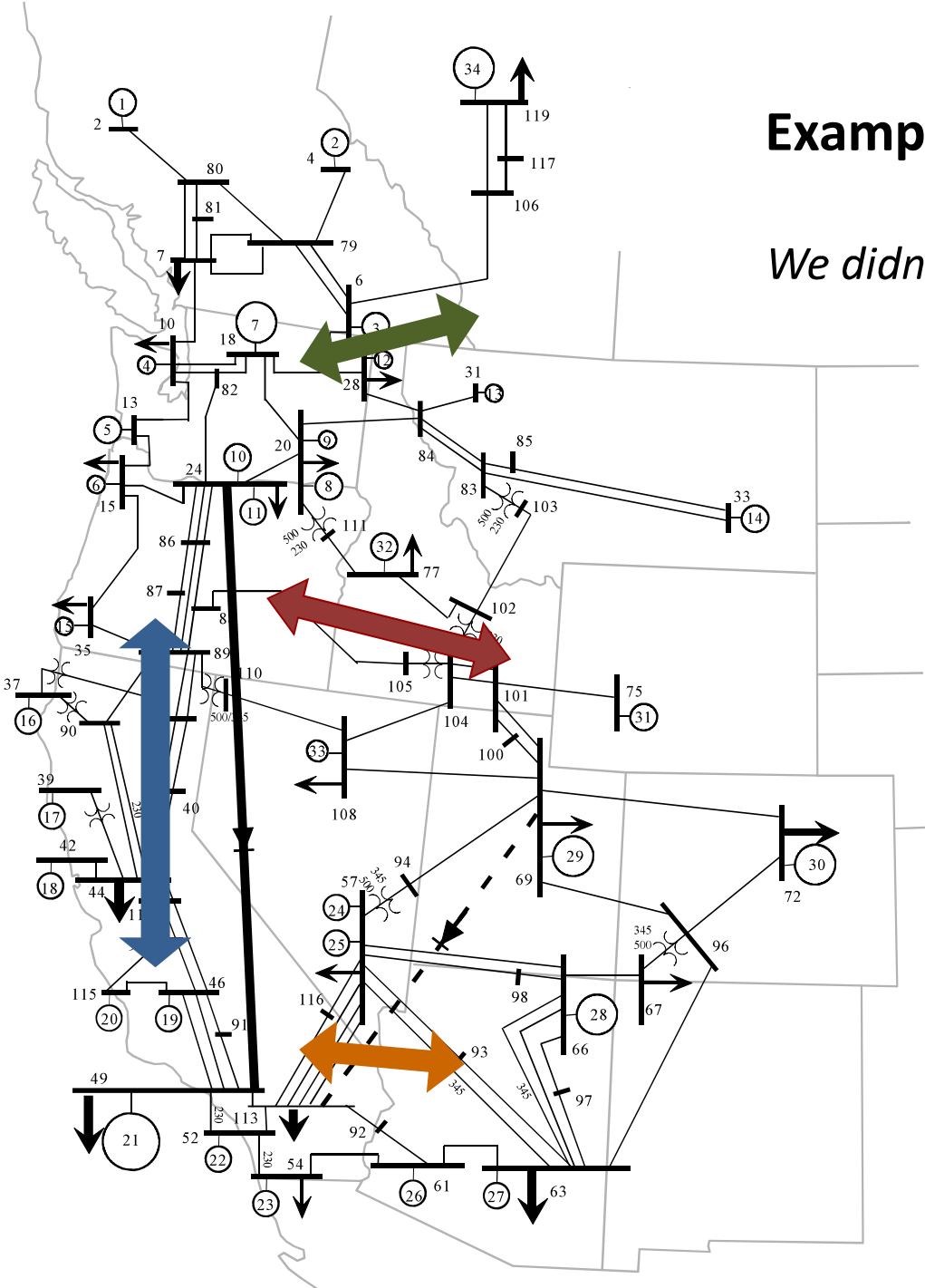


Generator swing equation

$$M \ddot{\delta} + D \dot{\delta} = P_m - P_e(\delta)$$

# Example: Characteristic oscillation modes

*We didn't know about these before PMUs...*



**Alberta**  
**0.4 Hz**

**East-West**  
**0.6 Hz**

**North-South**  
**0.25 Hz**

**California-Desert Southwest**  
**0.5 Hz**

## What if the signal is not strictly a cosine?

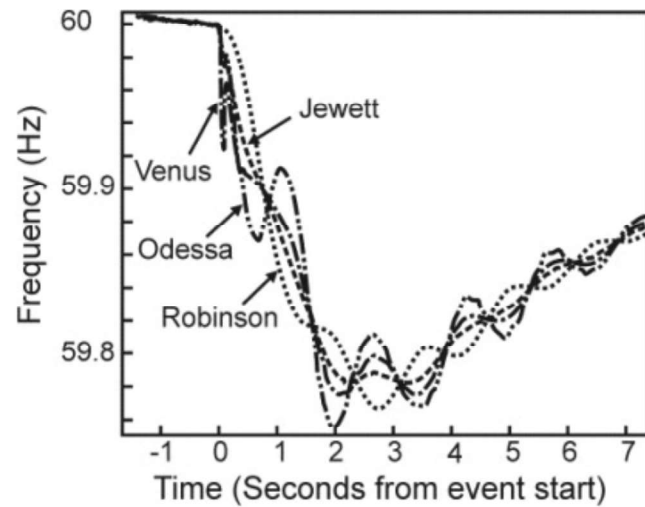


Figure 1 Observed frequency following generator loss

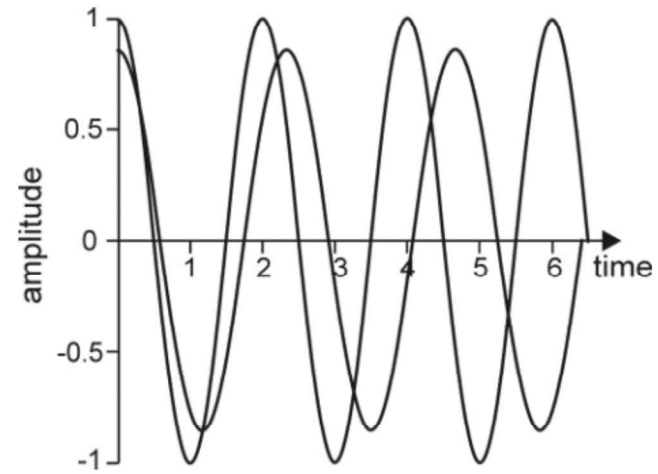
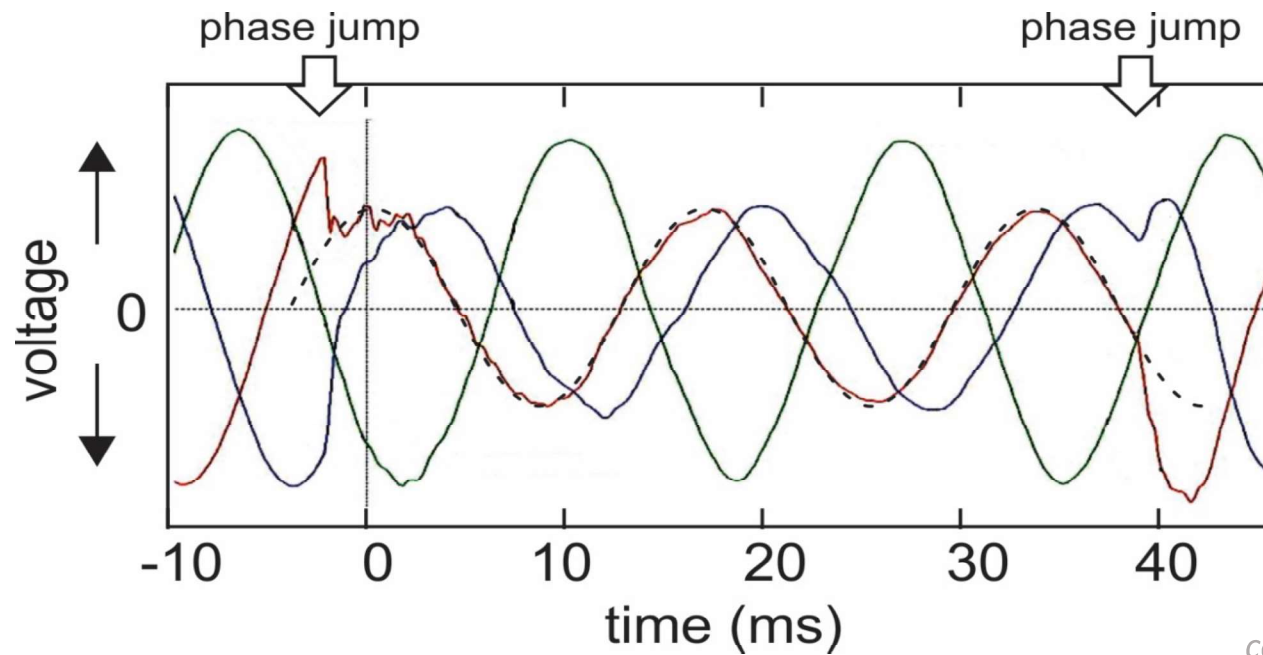


Figure 2. Two sine waves

*Harold Kirkham, PNNL: "The Measurand: The Problem of Frequency."*

*PMUs still give valuable insight, but their outputs are not obvious to define.*

## What if the signal is not strictly a cosine?



*courtesy of Harold Kirkham*

*The PMU answers the question,  
“If this signal were a cosine, what would its amplitude, frequency and phase be?”*

## What if the signal is not strictly a cosine?

$$x(t) = X_m \cos(\omega t + \varphi)$$

$$x(t) = X_m \cos \left\{ \left( \omega' + \frac{C_\omega}{2} t \right) t + \left( \varphi' + \frac{C_\varphi}{2} t \right) \right\}$$

*Allowing for  $\omega$  and  $\varphi$   
to vary in time*

$$x(t) = X_m \cos \left\{ \left( \omega' + \frac{C_\varphi}{2} + \frac{C_\omega}{2} t \right) t + \varphi' \right\}$$

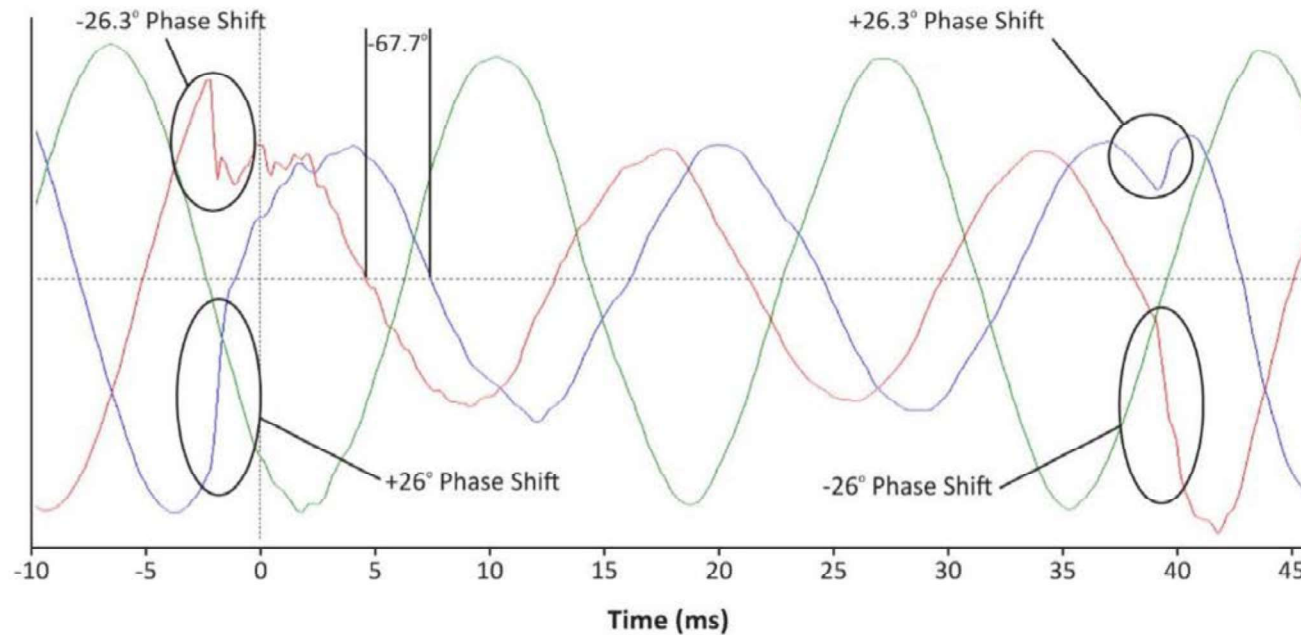
*Grouping terms*

*Harold Kirkham, PNNL:*

*"The Measurand: The Problem of Frequency."*

*There is more than one way to define frequency, phase, and rate of change of frequency (ROCOF).*

## Interesting times...



*Blue Cut Fire Incident, 2016:  
Inverters calculated frequency  
differently than might have been  
expected, and tripped offline.*

*NERC, "1,200 MW Fault Induced Solar  
Photovoltaic Resource Interruption Report,"  
June 2017*

*Observing and understanding the electric grid at increasingly higher resolution  
in space and time is increasingly important.*